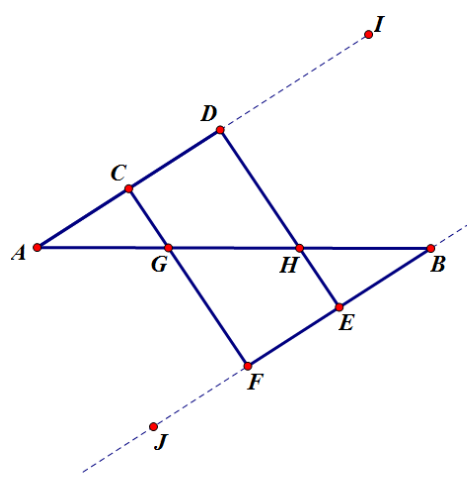
Trisect a Line Segment (5)

Method:

1) Construct a segment AB
2) Draw an arbitrary segment AI on point A but not-coincident with AB
3) Take an arbitrary point C on segment AI
4) Construct segment AC, then segment AD on segment AI such that AC = CD
5) Construct segment BJ on point B that is parallel to segment AI
6) Construct segments BE and EF on segment BJ such that BE = EF = AC = CD
7) Connect points C and F and call it segment CF
8) Connect points D and E and call it segment DE
9) Segments CF and DE intersects segment AB at G and H respectively.

Claim: Points G and H trisects segment AB.

Proof:



Consider ΔACG and ΔADH,

$\angle CAG = \angle DAH$ (Same angles)

AC = CD (By construction)

$\angle ACG = \angle ADH$ (CF and DE are transversals to parallel lines AI and BJ)

So, ΔACG and ΔADH are similar triangles.

Since, AC = CD, hence AG = GH

By similar argument we can show that ΔBEH and ΔBGF are similar triangles. Hence, BH = HG (as BE = EF)

Now for, ΔACG and ΔBEH

- $\angle A = \angle B$(AB is transversal to parallel lines AI and BJ)
- $AC = BE$ (by construction)
- $\angle ACG = \angle BEH$(CF and DE are transversals to parallel lines AI and BJ)

Hence, $\Delta ACG = \Delta BEH$ (A-A-S rule). So, AG = BH

Putting it all together, we have AG = GH = BH

Hence points G and H trisects segment AB.